

A CLASS OF PROCESSES ON THE PATH SPACE OVER A COMPACT RIEMANNIAN MANIFOLD WITH UNBOUNDED DIFFUSION

JÖRG-UWE LÖBUS

ABSTRACT. A class of diffusion processes on the path space over a compact Riemannian manifold is constructed. The diffusion of such a process is governed by an unbounded operator. A representation of the associated generator is derived and the existence of a certain local second moment is shown.

1. INTRODUCTION AND BASIC NOTATION

Infinite dimensional diffusion processes have been studied from several points of view. For example, S. Kusuoka [10], introduced diffusion type Dirichlet forms on Banach spaces. The existence of associated processes is then obtained by using regularity arguments. On the other hand, M. Röckner and T.S. Zhang [16] and A. Eberle [6] used finite dimensional approximation methods to treat infinite dimensional diffusion processes. In these papers, the diffusion is governed by bounded operators.

In contrast, we show the existence of a class of processes with unbounded diffusion operators. For this, we use methods and results of modern Dirichlet form theory (N. Bouleau and F. Hirsch [3], B.K. Driver and M. Röckner [5], M. Fukushima, Y. Oshima, and M. Takeda [8], Z.M. Ma and M. Röckner [13]). The basic structure of a diffusion form we deal with is

$$\mathcal{E}(F, F) := \int \langle \mathbf{D}F, \mathbf{A}\mathbf{D}F \rangle_{\mathbb{H}} d\nu, \quad F \in D(\mathcal{E}),$$

where \mathbb{H} is the Cameron-Martin space, \mathbf{D} denotes the corresponding gradient operator, and ν is the Wiener measure on the space $\mathbf{P}_{m_0}(M)$ of all Brownian paths γ on the compact Riemannian manifold M with $\gamma(0) = m_0 \in M$. In our setting, the diffusion operator $\mathbf{A} : L^2(\mathbf{P}_{m_0}(M) \rightarrow \mathbb{H}, \nu) \supseteq D(\mathbf{A}) \rightarrow L^2(\mathbf{P}_{m_0}(M) \rightarrow \mathbb{H}, \nu)$ is unbounded. Let us, however, mention that there are authors speaking in quite different situations of unbounded diffusion coefficients, namely when omitting the operator \mathbf{A} and replacing the measure $d\nu$ with $Cd\nu$ where C is a possibly unbounded density function (see, e.g., S. Aida [1]).

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We verify closability (Section 2) as well as quasi-regularity which implies the existence of an associated right process $\mathbf{M} = (\Omega, \mathcal{F}, (X_t)_{t \geq 0}, (P_\gamma)_{\gamma \in \mathbf{P}_{m_0}(M)})$ (Section 3). Furthermore, we provide a representation of the associated generator A (Section 4). In particular, the fact that a certain subspace of the space of the cylindrical functions over $\mathbf{P}_{m_0}(M)$ is a subset of the domain of A is used to determine the following local second moment,

$$\begin{aligned} \lim_{t \rightarrow 0} \frac{1}{t} \int \sum_{v=1}^N \|x^v(\gamma(\cdot)) - x^v(\tau(\cdot))\|_{L^2([0,1], ds)}^2 P_\tau(X_t \in d\gamma) \\ = \sum_{v=1}^N \int_{s \in [0,1]} \Gamma(x^v(\tau(s)), x^v(\tau(s))) ds, \end{aligned}$$

weakly in $L^1(\mathbf{P}_{m_0}(M), \nu)$ (Section 5). Here, M is considered as isometrically embedded in some \mathbb{R}^N and $x^v(p)$, $v \in \{1, \dots, N\}$, denote the standard coordinates of $p \in M$ embedded in \mathbb{R}^N ; finally, Γ is the carré du champ operator corresponding to $(\mathcal{E}, D(\mathcal{E}))$.

Let M be a compact connected Riemannian manifold of dimension d without boundary, isometrically embedded in some \mathbb{R}^N . Let T_m denote the tangent space to M at $m \in M$ and let $\langle \cdot, \cdot \rangle_{T_m}$ denote the inner product on T_m . We fix a covariant derivative ∇ compatible with the underlying Riemannian metric and assume that ∇ is torsion skew symmetric, which means that, if T is the torsion tensor of ∇ , then $\langle T(\xi, \eta), \eta \rangle \equiv 0$ for all vector fields ξ and η on M . This convention guarantees compatibility with the works of B.K. Driver [4], B.K. Driver and M. Röckner [5], and E.P. Hsu [9].

Let $O(M)$ denote the orthonormal frame bundle with respect to M . Furthermore, we denote the canonical projection $O(M) \rightarrow M$ by π and the canonical horizontal vector fields by H_1, \dots, H_d . Let X be the space of all Brownian trajectories on $[0, 1]$ with values in \mathbb{R}^d . For fixed $m_0 \in M$, we introduce the path space $\mathbf{P}_{m_0}(M)$ by

$$\mathbf{P}_{m_0}(M) := \{\gamma \in C([0, 1] \rightarrow M) : \gamma(0) = m_0\}$$

and equip it with the topology of uniform convergence. Let μ denote the Wiener measure on X and let $r_0 \in O(M)$ such that $\pi(r_0) = m_0$. According to J. Eells and D. Elworthy [7] and P. Malliavin [12], the solution r_x to the Stratonovich SDE

$$\begin{cases} \partial r_x(t) &= \sum_{i=1}^d H_i(r_x(t)) \partial x_i(t), \quad t \in [0, 1], \\ r_x(0) &= r_0, \end{cases}$$

$x = (x_1, \dots, x_d) \in X$, defines (μ -a.e.) a mapping $I : X \rightarrow \mathbf{P}_{m_0}(M)$ by $I(x)(t) := \pi(r_x(t))$, $x \in X$, $t \in [0, 1]$. Considering, simultaneously, x as a d -dimensional standard Brownian motion, $I(x)$ becomes a Brownian motion on M whose law on $\mathbf{P}_{m_0}(M)$ (the Wiener measure on $\mathbf{P}_{m_0}(M)$) is denoted by ν .

Finally, as discussed in [9], Section 4, there is an inverse map L of I in the sense that $L \circ I = \text{identity}$ μ -a.e. on X and $I \circ L = \text{identity}$ ν -a.e. on $\mathbf{P}_{m_0}(M)$. Note that, for $x \in X$ and $\gamma \in \mathbf{P}_{m_0}(M)$ with $\gamma = I(x)$ and $x = L(\gamma)$, the path r_x in $O(M)$ is well defined and that, for $a \in \mathbb{R}^d$ and $0 \leq s, t \leq 1$,

$$\langle r_x(s)a, r_x(s)a \rangle_{T_{\gamma(s)}} = \langle r_x(t)a, r_x(t)a \rangle_{T_{\gamma(t)}} = |a|_{\mathbb{R}^d}^2.$$

The parallel transport from $T_{\gamma(s)}$ to $T_{\gamma(t)}$ along $\gamma \in \mathbf{P}_{m_0}(M)$ is ν -a.e. defined as follows. For $x \in X$ and $\gamma \in \mathbf{P}_{m_0}(M)$ with $\gamma = I(x)$ and $x = L(\gamma)$, set

$$\mathcal{T}_{t \leftarrow s}^\gamma := r_x(t)r_x^{-1}(s), \quad 0 \leq s, t \leq 1.$$

Introduce the abbreviation $L^p(\nu)$ for $L^p(\mathbf{P}_{m_0}(M), \nu)$, $1 \leq p \leq \infty$, and define the set of all cylindrical functions on $\mathbf{P}_{m_0}(M)$,

$$Z := \left\{ F(\gamma) = f(\gamma(s_1); \dots; \gamma(s_k)), \gamma \in \mathbf{P}_{m_0}(M) : \right. \\ \left. 0 < s_1 < \dots < s_k = 1, f \in C^\infty(M^k), k \in \mathbb{N} \right\}.$$

Set

$$(1.1) \quad Y := \left\{ F(\gamma) = f(\gamma(s_1); \dots; \gamma(s_k)), \gamma \in \mathbf{P}_{m_0}(M) : \right. \\ \left. F \in Z, s_1, \dots, s_k \in \left\{ \frac{l}{2^n} : l \in \{1, \dots, 2^n\}, n \in \mathbb{N} \right\} \right\}.$$

As Z is dense in $L^2(\nu)$ (see [5]), Y is also dense in $L^2(\nu)$. Let $(e_j)_{j=1, \dots, d}$ be a standard basis in \mathbb{R}^d and let

$$H_1(t) = 1, \quad t \in [0, 1], \\ H_{2^m+k}(t) = \begin{cases} 2^{\frac{m}{2}} & \text{if } t \in [\frac{k-1}{2^m}, \frac{2k-1}{2^{m+1}}), \\ -2^{\frac{m}{2}} & \text{if } t \in [\frac{2k-1}{2^{m+1}}, \frac{k}{2^m}), \\ 0 & \text{otherwise,} \end{cases} \quad k = 1, \dots, 2^m, m = 0, 1, \dots,$$

denote the system of the Haar functions on $[0, 1]$. Furthermore, define

$$g_{d(r-1)+j} := H_r \cdot e_j, \quad r \in \mathbb{N}, j \in \{1, \dots, d\}.$$

As the system of the Haar functions $(H_n)_{n \in \mathbb{N}}$ is complete in $L^2([0, 1] \rightarrow \mathbb{R}, ds)$, the system $(g_n)_{n \in \mathbb{N}}$ is complete in $L^2([0, 1] \rightarrow \mathbb{R}^d, ds)$. Therefore, $(S_n)_{n \in \mathbb{N}}$, defined by

$$S_n(s) := \int_0^s g_n(u) du, \quad s \in [0, 1], n \in \mathbb{N},$$

is complete in the Cameron-Martin space \mathbb{H} , the space of all \mathbb{R}^d -valued absolutely continuous functions h on $[0, 1]$ with $h(0) = 0$ endowed with the norm

$$|h|_{\mathbb{H}} := \left(\int_0^1 |h'(s)|_{\mathbb{R}^d}^2 ds \right)^{\frac{1}{2}}.$$

2. DEFINITION OF THE FORM AND CLOSABILITY

For $F \in Y$ and ν -a.e. $\gamma \in \mathbf{P}_{m_0}(M)$, define

$$(2.1) \quad D_s F(\gamma) := \sum_{i=1}^k \chi_{[0, s_i]}(s) \{ \mathcal{T}_{s \leftarrow s_i}^\gamma (\nabla_{s_i} f)(\underline{\gamma}) \}, \quad s \in [0, 1],$$

where $(\nabla_{s_i} f)(\underline{\gamma}) \equiv (\nabla_{s_i} f)(\gamma(s_1); \dots; \gamma(s_k)) \in T_{\gamma(s_i)}$ denotes the gradient of the function f relative to the i -th variable while holding the remaining variables fixed.

Here, f and the s_1, \dots, s_k are as in the definition of Z . Furthermore, for $F \in Z$ and

$$\begin{aligned} \mathbf{D}_s F(\gamma) &:= \int_0^s r_{L(\gamma)}^{-1}(s') D_{s'} F(\gamma) ds' \\ (2.2) \quad &= \sum_{i=1}^k s \wedge s_i \cdot r_{L(\gamma)}^{-1}(s_i) (\nabla_{s_i} f)(\underline{\gamma}), \quad s \in [0, 1], \gamma \in \mathbf{P}_{m_0}(M), \end{aligned}$$

we have $\mathbf{D}F \in \mathbb{H}$ ν -a.e. See also (2.1).

Any $F \in Y$ has the representation $F(\gamma) = f(\gamma(s_1); \dots; \gamma(s_k))$ where $s_1 = \frac{l_1}{2^{n'}}, \dots, s_k = \frac{l_k}{2^{n'}}$, for some $k \in \mathbb{N}$, $n' \in \mathbb{N}$, $l_1, \dots, l_k \in \{1, \dots, 2^{n'}\}$, and $f \in C^\infty(M^k)$. As $H_{2^m+l}(t) = 0$ on $[0, 1] \setminus [\frac{l-1}{2^m}, \frac{l}{2^m}]$, $\int_{(l-1)/2^m}^{l/2^m} H_{2^m+l}(t) dt = 0$, and either $(\frac{l-1}{2^m}, \frac{l}{2^m}) \subseteq [0, s_i]$ or $(\frac{l-1}{2^m}, \frac{l}{2^m}) \subseteq (s_i, 1]$ if $m \geq n'$, $l \in \{1, \dots, 2^m\}$, and $i \in \{1, \dots, k\}$, from (2.2), we obtain the following lemma which is crucial for the technical procedure.

Lemma 2.1. *Let $F \in Y$. There exists $n_0 \in \mathbb{N}$ such that, for ν -a.e. $\gamma \in \mathbf{P}_{m_0}(M)$,*

$$\langle S_n, \mathbf{D}F(\gamma) \rangle_{\mathbb{H}} = 0, \quad n > n_0.$$

Let us define the diffusion operator we are dealing with in this paper. Choose an increasing sequence $(\lambda_n)_{n \in \mathbb{N}}$ of positive real numbers and define the operator

$$\begin{aligned} D(\mathbf{A}) &:= \left\{ \Phi \in L^2(\mathbf{P}_{m_0}(M) \rightarrow \mathbb{H}, \nu) : \int \sum_{i=1}^{\infty} \lambda_i^2 \langle S_i, \Phi \rangle_{\mathbb{H}}^2 d\nu < \infty \right\}, \\ \mathbf{A}\Phi(\gamma) &:= \sum_{i=1}^{\infty} \lambda_i \langle S_i, \Phi(\gamma) \rangle_{\mathbb{H}} S_i, \quad \gamma \in \mathbf{P}_{m_0}(M), \quad \Phi \in D(\mathbf{A}), \end{aligned}$$

mapping $L^2(\mathbf{P}_{m_0}(M) \rightarrow \mathbb{H}, \nu) \supseteq D(\mathbf{A}) \rightarrow L^2(\mathbf{P}_{m_0}(M) \rightarrow \mathbb{H}, \nu)$. For $F \in Y$, we have $\int \langle S_i, \mathbf{D}F \rangle_{\mathbb{H}}^2 d\nu < \infty$, $i \in \mathbb{N}$. By Lemma 2.1, for $F \in Y$, there is $n_0 \in \mathbb{N}$ such that, for all $i > n_0$, it holds that $\int \langle S_i, \mathbf{D}F \rangle_{\mathbb{H}}^2 d\nu = 0$. Therefore, we obtain $\{\mathbf{D}F : F \in Y\} \subseteq D(\mathbf{A})$. Furthermore, for all $F \in Y$, we get

$$\begin{aligned} \int \langle \mathbf{D}F, \mathbf{A}\mathbf{D}F \rangle_{\mathbb{H}} d\nu &= \int \left\langle \mathbf{D}F, \sum_{i=1}^{\infty} \lambda_i \langle S_i, \mathbf{D}F \rangle_{\mathbb{H}} S_i \right\rangle_{\mathbb{H}} d\nu \\ &= \sum_{i=1}^{\infty} \lambda_i \int \langle S_i, \mathbf{D}F \rangle_{\mathbb{H}}^2 d\nu \\ (2.3) \quad &< \infty. \end{aligned}$$

Consequently, the nonnegative symmetric bilinear form

$$\begin{aligned} \mathcal{E}(F, F) &:= \int \langle \mathbf{D}F, \mathbf{A}\mathbf{D}F \rangle_{\mathbb{H}} d\nu \\ (2.4) \quad &= \int \left| \mathbf{A}^{1/2} \mathbf{D}F \right|_{\mathbb{H}}^2 d\nu, \quad F \in Y, \end{aligned}$$

is well defined.

Remarks. (1) It is known from [5], Lemma 3, or [9], Proposition 5.3, that the operator $\mathbf{D} : Z \rightarrow L^2(\mathbf{P}_{m_0}(M) \rightarrow \mathbb{H}, \nu)$ is closable on $L^2(\nu)$. Let $(\mathbf{D}, D(\mathbf{D}))$ denote

this closure. Equivalently, the Ornstein-Uhlenbeck form

$$\mathcal{E}^{OU}(F, F) := \int |\mathbf{D}F|_{\mathbb{H}}^2 d\nu, \quad F \in Z,$$

is closable on $L^2(\nu)$. For its closure $(\mathcal{E}^{OU}, D(\mathcal{E}^{OU}))$, we have $D(\mathbf{D}) = D(\mathcal{E}^{OU})$.

(2) Fix $\gamma \in \mathbf{P}_{m_0}(M)$ and consider, for this remark, \mathbf{A} as an operator mapping $D(\mathbf{A}) \supseteq \mathbb{H} \rightarrow \mathbb{H}$. S. Albeverio and M. Röckner [2], for example, suggest in a similar situation to reduce a form of type (2.4) to a more simple one by choosing another Hilbert space $(H, \langle \cdot, \cdot \rangle_H) := (D(\mathbf{A}^{1/2}), \langle \mathbf{A}^{1/2} \cdot, \mathbf{A}^{1/2} \cdot \rangle_{\mathbb{H}})$. The price one has to pay is that the classical relation between directional derivative ∂_h and gradient \mathbf{D} , namely $\partial_h F = \langle \mathbf{D}F, h \rangle_H$, is in general not satisfied anymore if $\partial_h F = \langle \mathbf{D}F, h \rangle_{\mathbb{H}}$.

Theorem 2.2. *The bilinear form (\mathcal{E}, Y) is closable on $L^2(\nu)$.*

Proof. Suppose $F_n \in Y$, $n \in \mathbb{N}$, with $F_n \xrightarrow{n \rightarrow \infty} 0$ in $L^2(\nu)$ and $\mathcal{E}(F_n - F_m, F_n - F_m) \xrightarrow{m, n \rightarrow \infty} 0$. In particular, (2.4) implies

$$(2.5) \quad \mathbf{A}^{1/2} \mathbf{D}F_n \xrightarrow{n \rightarrow \infty} \Psi \quad \text{in } L^2(\mathbf{P}_{m_0}(M) \rightarrow \mathbb{H}, \nu)$$

for some $\Psi \in L^2(\mathbf{P}_{m_0}(M) \rightarrow \mathbb{H}, \nu)$. Define

$$(2.6) \quad \mathbf{J}F := \sum_{i=1}^{\infty} \lambda_i^{-1/2} \langle S_i, F \rangle_{\mathbb{H}} S_i, \quad F \in L^2(\mathbf{P}_{m_0}(M) \rightarrow \mathbb{H}, \nu).$$

Since \mathbf{J} is a bounded operator on $L^2(\mathbf{P}_{m_0}(M) \rightarrow \mathbb{H}, \nu)$, we verify

$$\mathbf{D}F_n = \mathbf{J} \mathbf{A}^{1/2} \mathbf{D}F_n \xrightarrow{n \rightarrow \infty} \mathbf{J}\Psi \quad \text{in } L^2(\mathbf{P}_{m_0}(M) \rightarrow \mathbb{H}, \nu)$$

from (2.5). As (\mathbf{D}, Z) is closable on $L^2(\nu)$, we obtain $\mathbf{J}\Psi = 0$. It follows from (2.6) and $\lambda_i > 0$, $i \in \mathbb{N}$, that $\Psi = 0$. Thus, relation (2.5) leads to $\mathbf{A}^{1/2} \mathbf{D}F_n \xrightarrow{n \rightarrow \infty} 0$ in $L^2(\mathbf{P}_{m_0}(M) \rightarrow \mathbb{H}, \nu)$ which implies $\mathcal{E}(F_n, F_n) = \int |\mathbf{A}^{1/2} \mathbf{D}F_n|_{\mathbb{H}}^2 d\nu \xrightarrow{n \rightarrow \infty} 0$. \square

Let $(\mathcal{E}, D(\mathcal{E}))$ denote the closure of (\mathcal{E}, Y) on $L^2(\nu)$.

Remark. (3) Let $F \in D(\mathcal{E})$ and let $F_n \in Y$, $n \in \mathbb{N}$, be a sequence converging to F in $\mathcal{E}_1^{1/2} = (\|\cdot\|_{L^2(\nu)} + \mathcal{E}(\cdot, \cdot))^{1/2}$ -norm. Since $\lambda_i > 0$, $i \in \mathbb{N}$, is an increasing sequence of positive real numbers and $(\mathcal{E}, Y) = (\mathcal{E}^{OU}, Y)$ if $\lambda_i = 1$, $i \in \mathbb{N}$, from (2.3) it follows that F_n , $n \in \mathbb{N}$, is a Cauchy sequence in $(\mathcal{E}^{OU})_1^{1/2} = (\|\cdot\|_{L^2(\nu)} + \mathcal{E}^{OU}(\cdot, \cdot))^{1/2}$ -norm. Therefore, $F_n \xrightarrow{n \rightarrow \infty} F$ in $(\mathcal{E}^{OU})_1^{1/2}$ -norm. Thus, we have $D(\mathcal{E}) \subseteq D(\mathcal{E}^{OU}) = D(\mathbf{D})$. Since, by self-adjointness, $\mathbf{A}^{1/2}$ is a closed operator, it holds that $\{\mathbf{D}F : F \in D(\mathcal{E})\} \subseteq D(\mathbf{A}^{1/2})$ and relations (2.3) and (2.4) yield

$$(2.7) \quad \begin{aligned} \mathcal{E}(F, F) &= \sum_{i=1}^{\infty} \lambda_i \int \langle S_i, \mathbf{D}F \rangle_{\mathbb{H}}^2 d\nu \\ &= \int |\mathbf{A}^{1/2} \mathbf{D}F|_{\mathbb{H}}^2 d\nu, \quad F \in D(\mathcal{E}). \end{aligned}$$

3. QUASI-REGULARITY AND ASSOCIATED PROCESS

Let $h \in \mathbb{H}$, $t \in \mathbb{R}$, $s \in [0, 1]$, and $\gamma \in \mathbf{P}_{m_0}(M)$ and let σ denote the solution to the geometric flow equation

$$\begin{cases} \dot{\sigma}^h(t, s)(\gamma) &= \mathcal{T}_{s \leftarrow 0}^{\sigma^h(t, \cdot)(\gamma)} r_0 h(s), \\ \sigma^h(0, s)(\gamma) &= \gamma(s). \end{cases}$$

Note that “ \cdot ” stands for differentiation with respect to t . In particular, we have $\sigma^h(\cdot, s)(\gamma) \in C^1(\mathbb{R} \rightarrow M)$, $\sigma^h(t)(\gamma) \equiv \sigma^h(t, \cdot)(\gamma) \in \mathbf{P}_{m_0}(M)$. For $h \in \mathbb{H}$ and ν -a.e. $\gamma \in \mathbf{P}_{m_0}(M)$, there exists a unique solution (see [4] and [9]). For $F \in Y$ given as in (1.1), the directional derivative along the direction $h \in \mathbb{H}$ satisfies

$$\begin{aligned} \partial_h F &:= \lim_{t \rightarrow 0} \frac{F(\sigma^h(t)) - F}{t} \\ &= \sum_{i=1}^k \langle \nabla_{s_i} f, \dot{\sigma}^h(0, s_i) \rangle_{T_{\cdot}(s_i)} \\ &= \sum_{i=1}^k \langle \nabla_{s_i} f, \mathcal{T}_{s_i \leftarrow 0} r_0 h(s_i) \rangle_{T_{\cdot}(s_i)} \\ &= \sum_{i=1}^k \langle \mathcal{T}_{0 \leftarrow s_i} (\nabla_{s_i} f), r_0 h(s_i) \rangle_{T_{m_0}} \\ (3.1) \quad &= \langle \mathbf{D}F, h \rangle_{\mathbb{H}} \quad \nu\text{-a.e.} \end{aligned}$$

See also (2.2).

Remark. (4) For every $h \in \mathbb{H}$, the operator $\partial_h : Z \rightarrow L^2(\nu)$ is closable on $L^2(\nu)$. Let $(\partial_h, D(\partial_h))$ denote the corresponding closure. It holds that $D(\mathcal{E}) \subseteq D(\mathbf{D}) \subseteq D(\partial_h)$, $h \in \mathbb{H}$, and $\partial_h F = \langle \mathbf{D}F, h \rangle_{\mathbb{H}}$, $F \in D(\mathbf{D})$; cf. [9], Theorem 5.2 and Proposition 5.3. Therefore,

$$\mathcal{E}(F, F) = \sum_{i=1}^{\infty} \lambda_i \int (\partial_{S_i} F)^2 d\nu, \quad F \in D(\mathcal{E}).$$

Proposition 3.1. *The form $(\mathcal{E}, D(\mathcal{E}))$ is a Dirichlet form on $L^2(\nu)$.*

Proof. We have

$$\begin{aligned} \mathcal{E}(F, F) &= \sum_{i=1}^{\infty} \lambda_i \int (\partial_{S_i} F)^2 d\nu \\ (3.2) \quad &= \sum_{i=1}^{\infty} \lambda_i \int \left(\frac{d}{dt} \Big|_0 F(\sigma^{S_i}(t)) \right)^2 d\nu, \quad F \in Y. \end{aligned}$$

It follows directly from [13], Proposition I, 4.10, and the chain rule that $(\mathcal{E}, D(\mathcal{E}))$ is a Dirichlet form. \square

An important tool for the subsequent technical procedure will be the following assertion; cf. [13], Chapter IV, Lemma 4.1. Note that, for $u, v \in D(\mathcal{E})$, we have $u \vee v \in D(\mathcal{E}) \subseteq D(\mathbf{D}) \subseteq D(\partial_{S_i})$, $i \in \mathbb{N}$ (see Remarks (3) and (4)).

Lemma 3.2. *Let $u, v \in D(\mathcal{E})$. For all $i \in \mathbb{N}$, we have*

$$\partial_{S_i}(u \vee v) = \chi_{\{u > v\}} \partial_{S_i} u + \chi_{\{u < v\}} \partial_{S_i} v + \frac{1}{2} \chi_{\{u=v\}} (\partial_{S_i} u + \partial_{S_i} v) \quad \nu\text{-a.e.}$$

Proof. Having representation (3.2) of (\mathcal{E}, Y) in mind, the proof can be obtained from that of [13], Chapter IV, Lemma 4.1 by replacing therein $\frac{\partial}{\partial k}$ with ∂_{S_i} and $\mathcal{F}C_b^\infty$ with Y . \square

Proposition 3.3. *Suppose*

$$(3.3) \quad \lambda_n \leq c n^{1-\varepsilon}, \quad n \in \mathbb{N}, \quad \text{for some } c > 0 \text{ and } \varepsilon \in (0, 1).$$

Then the Dirichlet form $(\mathcal{E}, D(\mathcal{E}))$ is quasi-regular.

Proof. In steps 1-3 below, we show that there is an \mathcal{E} -nest consisting of compact sets.

Step 1. For $r \in \mathbb{N}$, $l \in \{0, \dots, 2^{r-1} - 1\}$, and $k = 2^{r-1} + l$, set $s_k := (2l + 1)2^{-r}$. Let $x^v(p)$ denote the standard coordinates of $p \in M$ embedded in \mathbb{R}^N , $v \in \{1, \dots, N\}$. Fix $\tau \in \mathbf{P}_{m_0}(M)$, $k = 2^{r-1} + l$, and $v \in \{1, \dots, N\}$. Consider the functions $f_{v,k,\tau}(p) := x^v(p) - x^v(\tau(s_k))$, $p \in M$, and

$$(3.4) \quad F_{v,k,\tau}(\gamma) := f_{v,k,\tau}(\gamma(s_k)) = x^v(\gamma(s_k)) - x^v(\tau(s_k)), \quad \gamma \in \mathbf{P}_{m_0}(M);$$

obviously, $F_{v,k,\tau}$ belongs to Y . Furthermore, let either $i = j$ or $i = d(2^m + u - 1) + j$ for some $m \in \{0, 1, \dots\}$, $u \in \{1, \dots, 2^m\}$ and $j \in \{1, \dots, d\}$. We have

$$\begin{aligned} |\langle S_i, \mathbf{D}F_{v,k,\tau}(\gamma) \rangle_{\mathbb{H}}| &= \left| \langle T_{0 \leftarrow s_k}^\gamma \nabla_{s_k} x^v(\gamma(s_k)), r_0 S_i(s_k) \rangle_{T_{m_0}} \right| \\ &\leq \|T_{0 \leftarrow s_k}^\gamma \nabla_{s_k} x^v(\gamma(s_k))\|_{T_{m_0}} \|r_0 S_i(s_k)\|_{T_{m_0}} \\ &= \|\nabla_{s_k} x^v(\gamma(s_k))\|_{T_{\gamma(s_k)}} |S_i(s_k)|_{\mathbb{R}^d} \\ (3.5) \quad &= \|\nabla_{s_k} x^v(\gamma(s_k))\|_{T_{\gamma(s_k)}} |(S_i(s_k))^j| \end{aligned}$$

for ν -a.e. $\gamma \in \mathbf{P}_{m_0}(M)$. As mentioned in [5], proof of Proposition 5, $\|\nabla x^v(p)\|_{T_p}$, $p \in M$, is bounded by some constant K since M is compact. Furthermore, the definitions of s_k and S_i yield $|(S_i(s_k))^j| \leq 1$ if $i = j$ for some $j \in \{1, \dots, d\}$. Moreover, $|(S_i(s_k))^j| \leq 2^{-(m/2+1)}$ if $i = d(2^m + u - 1) + j$ for some $m \in \{0, 1, \dots\}$, $u \in \{1, \dots, 2^m\}$, $j \in \{1, \dots, d\}$, and $m < r$ as well as $s_k = (2l + 1)2^{-r} \in (\frac{u-1}{2^m}, \frac{u}{2^m})$. Otherwise, we have $S_i(s_k) = 0$. Therefore, (3.5) implies

$$\begin{aligned} \sum_{i=1}^{\infty} \lambda_i (\partial_{S_i} F_{v,k,\tau}(\gamma))^2 &= \sum_{i=1}^{\infty} \lambda_i \langle S_i, \mathbf{D}F_{v,k,\tau}(\gamma) \rangle_{\mathbb{H}}^2 \\ &= \sum_{j=1}^d \lambda_j \langle S_j, \mathbf{D}F_{v,k,\tau}(\gamma) \rangle_{\mathbb{H}}^2 \\ &\quad + \sum_{m=0}^{\infty} \sum_{u=1}^{2^m} \sum_{j=1}^d \lambda_{d(2^m+u-1)+j} \langle S_{d(2^m+u-1)+j}, \mathbf{D}F_{v,k,\tau}(\gamma) \rangle_{\mathbb{H}}^2 \\ (3.6) \quad &\leq K^2 d \lambda_d + K^2 d \sum_{m=0}^{\infty} \lambda_{d 2^{m+1}} 2^{-(m+2)} \end{aligned}$$

for ν -a.e. $\gamma \in \mathbf{P}_{m_0}(M)$. Finally, from (3.3), we obtain

$$(3.7) \quad \sum_{i=1}^{\infty} \lambda_i (\partial_{S_i} F_{v,k,\tau}(\gamma))^2 \leq K^2 c d^{2-\varepsilon} \frac{2^{1+\varepsilon} - 1}{2^{1+\varepsilon} - 2} =: C_1$$

for ν -a.e. $\gamma \in \mathbf{P}_{m_0}(M)$ and

$$(3.8) \quad \mathcal{E}(F_{v,k,\tau}, F_{v,k,\tau}) \leq C_1,$$

where the right-hand side is independent of k (resp. s_k), $v \in \{1, \dots, N\}$, and τ .

Step 2. We apply a method introduced in [5] and [15]. Set

$$G_{n,\tau} := \sup_{\substack{k \in \{1, \dots, n\} \\ v \in \{1, \dots, N\}}} |F_{v,k,\tau}|, \quad n \in \mathbb{N}.$$

It follows now from (3.7), relation

$$\mathcal{E}(G_{n,\tau}, G_{n,\tau}) = \int \sum_{i=1}^{\infty} \lambda_i (\partial_{S_i} G_{n,\tau}(\gamma))^2 d\nu,$$

and Lemma 3.2 that

$$\mathcal{E}(G_{n,\tau}, G_{n,\tau}) \leq C_1, \quad n \in \mathbb{N}.$$

Since M is compact, there exists $C_2 > 0$, such that

$$|x^v(p)| \leq \frac{1}{2} \sqrt{C_2}, \quad p \in M, \quad v \in \{1, \dots, N\}.$$

From (3.4) and the definition of $G_{n,\tau}$ it follows that $\|G_{n,\tau}\|_{L^2(\nu)}^2 \leq C_2$ and, thus,

$$(3.9) \quad \mathcal{E}_1(G_{n,\tau}, G_{n,\tau}) \leq C_1 + C_2 =: C_3, \quad n \in \mathbb{N}.$$

Step 3. In this step, we proceed as in [5] and [15]. In particular, we apply the Banach-Saks property of the Hilbert space $(D(\mathcal{E}), \mathcal{E}_1^{1/2})$, which states that every bounded sequence in $(D(\mathcal{E}), \mathcal{E}_1^{1/2})$ has a subsequence whose Cesaro means converge strongly (see, for example, [14]). Accordingly, relation (3.9) and the fact that the sequence $(G_{n,\tau})_{n \in \mathbb{N}}$ satisfies $G_{n,\tau} \leq G_{n+1,\tau}$, $n \in \mathbb{N}$, imply that the function

$$H_\tau(\gamma) := \sup_{\substack{s \in [0,1] \\ v \in \{1, \dots, N\}}} |x^v(\gamma(s)) - x^v(\tau(s))|, \quad \gamma \in \mathbf{P}_{m_0}(M),$$

belongs to $D(\mathcal{E})$ and that

$$\mathcal{E}_1(H_\tau, H_\tau) \leq C_3.$$

Let $\{\tau_k : k \in \mathbb{N}\}$ be a dense set in $\mathbf{P}_{m_0}(M)$. Set

$$K_n := \inf_{1 \leq k \leq n} H_{\tau_k}, \quad n \in \mathbb{N}.$$

We have $K_n \in D(\mathcal{E})$. Again, recalling the Banach-Saks property of $(D(\mathcal{E}), \mathcal{E}_1^{1/2})$, the last relation implies that $\mathcal{E}_1(K_n, K_n) \xrightarrow{n \rightarrow \infty} 0$. According to [13], Chapter III, Proposition 3.5, there exists a subsequence K_{n_k} , $k \in \mathbb{N}$, and an \mathcal{E} -nest F_m , $m \in \mathbb{N}$, such that K_{n_k} converges uniformly to zero (as $k \rightarrow \infty$) on each F_m . Consult also [3], Section I.8. As in [5], proof of Proposition 5, it follows now from the definition of K_n , $n \in \mathbb{N}$, that each F_m is totally bounded. Thus, F_m , $m \in \mathbb{N}$, form an \mathcal{E} -nest consisting of compact sets.

Step 4. For fixed $\tau \in \mathbf{P}_{m_0}(M)$, the system of functions $F_{v,k,\tau}$, $v \in \{1, \dots, N\}$, $k \in \mathbb{N}$, introduced in (3.4) separates the points in $\mathbf{P}_{m_0}(M)$.

Together with Theorem 2.2 and the result of Step 3, quasi-regularity follows now from its definition (see [13], Chapter IV, Definition 3.1). \square

Proposition 3.4. *The form $(\mathcal{E}, D(\mathcal{E}))$ is local.*

Proof. We follow [5], proof of Proposition 5 (ii), and [13], Example V.1.12. Let $F, G \in D(\mathcal{E}) \cap L^\infty(\nu)$ with $\text{supp}[F] \cap \text{supp}[G] = \emptyset$. According to [13], Propositions I.4.17 (i) and V.1.2 (ii), we have to verify $\mathcal{E}(F, G) = 0$. Since $\mathcal{E}(F, G) = \int \langle \mathbf{A}^{1/2} \mathbf{D}F, \mathbf{A}^{1/2} \mathbf{D}G \rangle_{\mathbb{H}} d\nu$ (cf. (2.7)), it is sufficient to show that

$$(3.10) \quad \mathbf{D}F = 0 \quad \nu\text{-a.e. on } \mathbf{P}_{m_0}(M) \setminus \text{supp}[F].$$

From $D(\mathcal{E}) \subseteq D(\mathcal{E}^{OU})$ and [5], equation (11), we obtain

$$(3.11) \quad \mathbf{D}(U \cdot V) = U \cdot \mathbf{D}(V) + V \cdot \mathbf{D}(U), \quad U, V \in D(\mathcal{E}) \cap L^\infty(\nu).$$

See also [13], Example V.1.12. Furthermore, from [13], Proposition V.1.7, we get the existence of $V \in D(\mathcal{E}) \cap L^\infty(\nu)$ with $0 \leq V \leq \chi_{\mathbf{P}_{m_0}(M) \setminus \text{supp}[F]}$ and $V > 0$ ν -a.e. on $\mathbf{P}_{m_0}(M) \setminus \text{supp}[F]$; here χ denotes the indicator function. Now, relation (3.11) implies

$$0 = F \cdot \mathbf{D}(V) + V \cdot \mathbf{D}(F) \quad \nu\text{-a.e.}$$

This yields (3.10). \square

As a consequence of Propositions 3.3 and 3.4, we get with [13], Theorems IV.3.5 and V.1.11:

Theorem 3.5. *There exists a diffusion process \mathbf{M} associated with $(\mathcal{E}, D(\mathcal{E}))$.*

4. GENERATOR

We start with a technical lemma.

Lemma 4.1. *Let $F \in Y$ and $n \in \mathbb{N}$. Then the derivatives $\frac{d}{dt}\big|_0 \partial_{S_n} F(\sigma^{S_n}(-t))$, $\frac{d}{dt}\big|_0 \frac{d\nu \circ \sigma^{S_n}(-t)}{d\nu}$, and $\frac{d}{dt}\big|_0 \left\{ \partial_{S_n} F(\sigma^{S_n}(-t)) \frac{d\nu \circ \sigma^{S_n}(-t)}{d\nu} \right\}$ exist in $L^2(\nu)$ and we have*

$$(4.1) \quad \begin{aligned} \frac{d}{dt}\bigg|_0 \left\{ \partial_{S_n} F(\sigma^{S_n}(-t)) \frac{d\nu \circ \sigma^{S_n}(-t)}{d\nu} \right\} \\ = -\partial_{S_n} \partial_{S_n} F + \frac{d}{dt}\bigg|_0 \frac{d\nu \circ \sigma^{S_n}(-t)}{d\nu} \cdot \partial_{S_n} F \quad \nu\text{-a.e.} \end{aligned}$$

Proof. Step 1. Introduce

$$\varphi_t(\gamma) := \frac{d\nu \circ \sigma^{S_n}(-t)(\gamma)}{d\nu}(\gamma), \quad \gamma \in \mathbf{P}_{m_0}(M), \quad t \in \mathbb{R}.$$

The existence of $\frac{d}{dt}\big|_0 \varphi_t$ in $L^2(\nu)$ is shown in [4], Theorem 8.5 and in the proof of Theorem 9.1. Note that the result for $h \in C^1([0, 1] \rightarrow \mathbb{R}^d)$ presented in [4] can be extended to general $h \in \mathbb{H}$ by [9], Theorems 3.5 and 4.1 and the proof of Theorem 5.1.

Step 2. Let $n \in \mathbb{N}$ and let $F \in Y$ be given as in (1.1). According to (3.1), we have, for ν -a.e. $\gamma \in \mathbf{P}_{m_0}(M)$,

$$\begin{aligned} \psi_t(\gamma) &:= \partial_{S_n} F(\sigma^{S_n}(-t)(\gamma)) \\ &= \sum_{i=1}^k \left\langle T_{0 \leftarrow s_i}^{\sigma^{S_n}(-t)(\gamma)}(\nabla_{s_i} f)(\sigma^{S_n}(-t)(\gamma)), r_0 S_n(s_i) \right\rangle_{T_{m_0}}, \quad t \in \mathbb{R}. \end{aligned}$$

From $|S_n(s)|_{\mathbb{R}^d} \leq 1$, $s \in [0, 1]$, it follows that, for ν -a.e. $\gamma \in \mathbf{P}_{m_0}(M)$,

$$\begin{aligned} |\psi_t(\gamma)| &\leq \sum_{i=1}^k \left\| \mathcal{T}_{0 \leftarrow s_i}^{\sigma^{S_n}(-t)(\gamma)} (\nabla_{s_i} f) (\underline{\sigma^{S_n}(-t)(\gamma)}) \right\|_{T_{m_0}} \|r_0 S_n(s_i)\|_{T_{m_0}} \\ &\leq \sum_{i=1}^k \left\| (\nabla_{s_i} f) (\underline{\sigma^{S_n}(-t)(\gamma)}) \right\|_{T_{\sigma^{S_n}(-t, s_i)(\gamma)}}, \quad t \in \mathbb{R}. \end{aligned}$$

Since $f \in C^\infty(M^k)$, and M is compact, there exists $C_4 > 0$ such that

$$(4.2) \quad |\psi_t(\gamma)| \leq C_4, \quad \nu\text{-a.e. } \gamma \in \mathbf{P}_{m_0}(M), \quad t \in \mathbb{R}.$$

Furthermore, in virtue of [9], Theorem 4.1 (iii),

$$(4.3) \quad \psi_t \xrightarrow[t \rightarrow 0]{} \psi_0 \quad \nu\text{-a.e.}$$

Step 3. The aim of this step is to verify the existence of $\frac{d}{dt}|_0 \psi_t$ in $L^2(\nu)$. To this end, fix $i \in \{1, \dots, k\}$. Since $(\nabla_{s_i} f)^v$ is then a smooth function on M^k , from [9], Section 5, and [4], Lemma 9.1, it follows that

$$(4.4) \quad \frac{d}{dt} \Big|_0 \left((\nabla_{s_i} f) (\underline{\sigma^{S_n}(-t)(\gamma)}) \right)^v \text{ exists in } L^2(\nu), \quad v \in \{1, \dots, N\}.$$

Furthermore, there is a $C_5 > 0$ such that

$$(4.5) \quad |(\nabla_{s_i} f)^v| \leq C_5, \quad v \in \{1, \dots, N\}.$$

By [4], Corollary 4.2 and inequalities (i) as well as (ii) of Lemma 4.1 of the same reference, for all $v \in \{1, \dots, N\}$,

$$\begin{aligned} (4.6) \quad \frac{d}{dt} \Big|_0 \left(\mathcal{T}_{s_i \leftarrow 0}^{\sigma^{S_n}(-t)(\gamma)} r_0 S_n(s_i) \right)^v \\ = \frac{d}{dt} \Big|_0 \left(r_{L(\gamma(-t, j, i))}(s_i) S_n(s_i) \right)^v \text{ exists in } L^2(\nu) \end{aligned}$$

if $n = j \in \{1, \dots, d\}$. Even though in [4] the geometric flow is generated by a C^1 -function, for $n = d(2^m + l - 1) + j$ with $m \in \{0, 1, \dots\}$, $l \in \{1, \dots, 2^m\}$, $j \in \{1, \dots, d\}$, we may obtain (4.6) from the above reference by decomposing $S_n = S_n^1 + S_n^2 + S_n^3$ where

$$\begin{aligned} S_n^1 &= 2^{\frac{m}{2}} \chi_{[\frac{l-1}{2^m}, 1]}(s) \left(s - \frac{l-1}{2^m} \right), \\ S_n^2 &= -2^{\frac{m}{2}+1} \chi_{[\frac{2l-1}{2^{m+1}}, 1]}(s) \left(s - \frac{2l-1}{2^{m+1}} \right), \\ S_n^3 &= 2^{\frac{m}{2}} \chi_{[\frac{l}{2^m}, 1]}(s) \left(s - \frac{l}{2^m} \right). \end{aligned}$$

Moreover, by isometric embedding of M into \mathbb{R}^N we verify

$$\begin{aligned} \left| \left(\mathcal{T}_{s_i \leftarrow 0} r_0 S_n(s_i) \right)^v \right| &\leq \left| \mathcal{T}_{s_i \leftarrow 0} r_0 S_n(s_i) \right|_{\mathbb{R}^N} \\ &= \left\| \mathcal{T}_{s_i \leftarrow 0} r_0 S_n(s_i) \right\|_{T_{\cdot}(s_i)} \\ &= |S_n(s_i)|_{\mathbb{R}^d} \\ (4.7) \quad &\leq 1, \quad v \in \{1, \dots, N\}. \end{aligned}$$

Introduce

$$\alpha_t^v := \left((\nabla_{s_i} f) (\underline{\sigma^{S_n}(-t)(\gamma)}) \right)^v, \quad \beta_t^v := \left(\mathcal{T}_{s_i \leftarrow 0}^{\sigma^{S_n}(-t)(\gamma)} r_0 S_n(s_i) \right)^v,$$

$t \in \mathbb{R}$, $v \in \{1, \dots, N\}$. The existence of

$$\frac{d}{dt} \Big|_0 \sum_{v=1}^N ((\nabla_{s_i} f)(\sigma^{S_n}(-t)(\gamma)))^v (\mathcal{T}_{s_i \leftarrow 0}^{\sigma^{S_n}(-t)(\gamma)} r_0 S_n(s_i))^v = \frac{d}{dt} \Big|_0 \sum_{v=1}^N \alpha_t^v \beta_t^v$$

in $L^2(\nu)$ follows from (4.4)-(4.7), $\alpha_t^v \xrightarrow{t \rightarrow 0} \alpha_0^v$ ν -a.e., and the inequality

$$\begin{aligned} & \left\| \frac{\sum_{v=1}^N \alpha_t^v \beta_t^v - \sum_{v=1}^N \alpha_0^v \beta_0^v}{t} - \sum_{v=1}^N (\dot{\alpha}_0^v \beta_0^v + \alpha_0^v \dot{\beta}_0^v) \right\|_{L^2(\nu)} \\ & \leq \sum_{v=1}^N \left\| \left(\frac{\alpha_t^v - \alpha_0^v}{t} - \dot{\alpha}_0^v \right) \beta_0^v \right\|_{L^2(\nu)} + \sum_{v=1}^N \left\| \left(\frac{\beta_t^v - \beta_0^v}{t} - \dot{\beta}_0^v \right) \alpha_t^v \right\|_{L^2(\nu)} \\ & \quad + \sum_{v=1}^N \left\| (\alpha_t^v - \alpha_0^v) \dot{\beta}_0^v \right\|_{L^2(\nu)} \\ & \leq \sum_{v=1}^N \left\| \frac{\alpha_t^v - \alpha_0^v}{t} - \dot{\alpha}_0^v \right\|_{L^2(\nu)} \|\beta_0^v\|_{L^\infty(\nu)} + C_5 \sum_{v=1}^N \left\| \frac{\beta_t^v - \beta_0^v}{t} - \dot{\beta}_0^v \right\|_{L^2(\nu)} \\ & \quad + \sum_{v=1}^N \left\| ((\alpha_t^v - \alpha_0^v) \dot{\beta}_0^v)^2 \right\|_{L^1(\nu)}^{1/2}, \quad v \in \{1, \dots, N\}; \end{aligned}$$

note that, by (4.5), $((\alpha_t^v - \alpha_0^v) \dot{\beta}_0^v)^2 \in L^1(\nu)$ is dominated by $4C_5^2 (\dot{\beta}_0^v)^2 \in L^1(\nu)$, $v \in \{1, \dots, N\}$. Finally, by isometry

$$\frac{d}{dt} \Big|_0 \psi_t(\gamma) = \sum_{i=1}^k \frac{d}{dt} \Big|_0 \left\langle (\nabla_{s_i} f)(\sigma^{S_n}(-t)(\gamma)), \mathcal{T}_{s_i \leftarrow 0}^{\sigma^{S_n}(-t)(\gamma)} r_0 S_n(s_i) \right\rangle_{T_{\sigma^{S_n}(-t, s_i)}(\gamma)}$$

exists $L^2(\nu)$.

Step 4. Having in mind that $\varphi_0 \equiv 1$ and that $((\psi_t - \psi_0) \dot{\varphi}_0)^2 \in L^1(\nu)$ is dominated by $4C_4^2 (\dot{\varphi}_0)^2 \in L^1(\nu)$, the existence of $\frac{d}{dt} \Big|_0 \varphi_t$ and $\frac{d}{dt} \Big|_0 \psi_t$ in $L^2(\nu)$ (cf. Steps 1 and 3), relations (4.2), (4.3), and

$$\begin{aligned} & \left\| \frac{\varphi_t \psi_t - \varphi_0 \psi_0}{t} - \dot{\varphi}_0 \psi_0 - \varphi_0 \dot{\psi}_0 \right\|_{L^2(\nu)} \\ & \leq C_4 \left\| \frac{\varphi_t - \varphi_0}{t} - \dot{\varphi}_0 \right\|_{L^2(\nu)} + \left\| \frac{\psi_t - \psi_0}{t} - \dot{\psi}_0 \right\|_{L^2(\nu)} + \|((\psi_t - \psi_0) \dot{\varphi}_0)^2\|_{L^1(\nu)}^{1/2} \end{aligned}$$

imply the existence of $\frac{d}{dt} \Big|_0 \left\{ \partial_{S_n} F(\sigma^{S_n}(-t)) \frac{d\nu \circ \sigma^{S_n}(-t)}{d\nu} \right\}$ in $L^2(\nu)$.

Step 5. As shown, for example in [11], 4.1, it holds that $\partial_{S_n} \eta = \frac{d}{dt} \Big|_0 \eta(\sigma^{S_n}(-t))$, whenever $\eta \in D(\mathbf{D}) \subseteq D(\partial_{S_n})$ and the limit $\lim_{t \rightarrow 0} (\eta(\sigma^{S_n}(t)) - \eta)$ exists in $L^2(\nu)$. Since $F \in Y$ belongs to $D(\mathbf{D}^2)$ (see [9], Section 6), we have $\partial_{S_n} F = (\mathbf{D}_1 F)^j \in D(\mathbf{D})$ if $n = j \in \{1, \dots, d\}$ and we have $\partial_{S_n} F = 2^{m/2} (-\mathbf{D}_{\frac{l-1}{2^m}} F + 2\mathbf{D}_{\frac{2l-1}{2^{m+1}}} F - \mathbf{D}_{\frac{l}{2^m}} F)^j \in D(\mathbf{D})$ if $n = d(2^m + l - 1) + j$ for $m \in \{0, 1, \dots\}$, $l \in \{1, \dots, 2^m\}$, $j \in \{1, \dots, d\}$. Therefore, relation (4.1) is a consequence of Step 4, in particular, of (4.8). \square

Let $(A, D(A))$ denote the generator of $(\mathcal{E}, D(\mathcal{E}))$. Fix a version \bar{H} of the map $\gamma \rightarrow \mathcal{T}_{\leftarrow 0}^\gamma$. Using B.K. Driver's geometrical notation (see [4], Definition 6.2 and

Theorem 9.1), we introduce

$$z_n(\gamma) = \frac{1}{2} \int_0^1 \left(Ric_{\bar{H}\gamma} \langle S_n \rangle + \hat{\Theta}_{\bar{H}\gamma} \langle S_n \rangle \right) \cdot dx + \int_0^1 S'_n \cdot dx, \quad n \in \mathbb{N},$$

where $\gamma = I(x) \in \mathbf{P}_{m_0}(M)$ for μ -a.e. $x \in X$.

Theorem 4.2. *Let $F \in Y$. We have $F \in D(A)$ and*

$$AF = \sum_{n=1}^{\infty} \lambda_n \{ \partial_{S_n} \partial_{S_n} F + z_n \partial_{S_n} F \} \quad \nu\text{-a.e.}$$

Proof. Let $G \in Y$. From (3.2), we obtain

$$(4.9) \quad \mathcal{E}(F, G) = \int \sum_{n=1}^{\infty} \lambda_n \left. \frac{d}{dt} \right|_0 G(\sigma^{S_n}(t)) \partial_{S_n} F \, d\nu.$$

Taking into consideration that $\left. \frac{d}{dt} \right|_0 G(\sigma^{S_n}(t))$ exists in $L^2(\nu)$, that $\partial_{S_n} F = \langle S_n, \mathbf{D}F \rangle_{\mathbb{H}} \in L^2(\nu)$, and that the sum in (4.9) is, by Lemma 2.1, actually a finite one, we obtain

$$\mathcal{E}(F, G) = \left. \frac{d}{dt} \right|_0 \int \sum_{n=1}^{\infty} \lambda_n G(\sigma^{S_n}(t)) \partial_{S_n} F \, d\nu.$$

Under the substitution $\gamma \rightarrow \sigma^{S_n}(-t)(\gamma)$, we get

$$\begin{aligned} \mathcal{E}(F, G) &= \left. \frac{d}{dt} \right|_0 \int G \cdot \sum_{n=1}^{\infty} \lambda_n \partial_{S_n} F(\sigma^{S_n}(-t)) \, d\nu \circ \sigma^{S_n}(-t) \\ &= \left. \frac{d}{dt} \right|_0 \int G \cdot \sum_{n=1}^{\infty} \lambda_n \partial_{S_n} F(\sigma^{S_n}(-t)) \frac{d\nu \circ \sigma^{S_n}(-t)}{d\nu} \, d\nu. \end{aligned}$$

From Lemma 4.1, it can be concluded that

$$(4.10) \quad \mathcal{E}(F, G) = \int G \cdot \sum_{n=1}^{\infty} \lambda_n \left. \frac{d}{dt} \right|_0 \left\{ \partial_{S_n} F(\sigma^{S_n}(-t)) \frac{d\nu \circ \sigma^{S_n}(-t)}{d\nu} \right\} \, d\nu;$$

again, take into consideration that the above sum is actually a finite one. Let G^* be an arbitrary function belonging to $D(\mathcal{E})$. As Y is by Theorem 2.2 dense in $D(\mathcal{E})$ with respect to $\mathcal{E}_1^{1/2}$ -norm, we find a sequence $G_m \in Y$, $m \in \mathbb{N}$, with $G_m \xrightarrow{m \rightarrow \infty} G^*$ in $\mathcal{E}_1^{1/2}$ -norm. In particular, $G_m \xrightarrow{m \rightarrow \infty} G^*$ in $L^2(\nu)$. On account of $|\mathcal{E}(G_m - G^*, F) + (G_m - G^*, F)_{L^2(\nu)}| = |\mathcal{E}_1(G_m - G^*, F)| \xrightarrow{m \rightarrow \infty} 0$, we verify $\mathcal{E}(G_m, F) \xrightarrow{m \rightarrow \infty} \mathcal{E}(G^*, F)$. Now, relation (4.10) and Lemma 4.1 yield

$$\mathcal{E}(F, G^*) = \int G^* \cdot \sum_{n=1}^{\infty} \lambda_n \left. \frac{d}{dt} \right|_0 \left\{ \partial_{S_n} F(\sigma^{S_n}(-t)) \frac{d\nu \circ \sigma^{S_n}(-t)}{d\nu} \right\} \, d\nu.$$

Therefore, we have $F \in D(A)$ and

$$AF = - \sum_{n=1}^{\infty} \lambda_n \left. \frac{d}{dt} \right|_0 \left\{ \partial_{S_n} F(\sigma^{S_n}(-t)) \frac{d\nu \circ \sigma^{S_n}(-t)}{d\nu} \right\} \quad \nu\text{-a.e.}$$

From Lemma 4.1, it can be deduced that

$$(4.11) \quad AF = \sum_{n=1}^{\infty} \lambda_n \left\{ \partial_{S_n} \partial_{S_n} F - \frac{d}{dt} \Big|_0 \frac{d\nu \circ \sigma^{S_n}(-t)}{d\nu} \cdot \partial_{S_n} F \right\} \quad \nu\text{-a.e.}$$

According to [4], Lemma 9.2, or [9], proof of Theorem 5.1, it holds that

$$\frac{d}{dt} \Big|_0 \frac{d\nu \circ \sigma^{S_n}(-t)}{d\nu} = -z_n \quad \nu\text{-a.e.}$$

From (4.11), it follows that

$$AF = \sum_{n=1}^{\infty} \lambda_n \{ \partial_{S_n} \partial_{S_n} F + z_n \partial_{S_n} F \} \quad \nu\text{-a.e.}$$

□

Remark. (5) Keeping in mind that, for $F \in Y$, the sum $\sum_{n=1}^{\infty} \lambda_n z_n \partial_{S_n} F$ is actually a finite sum (cf. Lemma 2.1), we can verify the following identity:

$$\begin{aligned} & \sum_{n=1}^{\infty} \lambda_n z_n \partial_{S_n} F(\gamma) \\ &= \sum_{n=1}^{\infty} \lambda_n \left\{ \frac{1}{2} \int_0^1 \left(Ric_{\bar{H}\gamma} \langle S_n \rangle + \hat{\Theta}_{\bar{H}\gamma} \langle S_n \rangle \right) \cdot dx + \int_0^1 S'_n \cdot dx \right\} \partial_{S_n} F(\gamma) \\ &= \frac{1}{2} \int_0^1 \left(Ric_{\bar{H}\gamma} \langle \sum \lambda_n \partial_{S_n} F S_n \rangle + \hat{\Theta}_{\bar{H}\gamma} \langle \sum \lambda_n \partial_{S_n} F S_n \rangle \right) \cdot dx \\ &\quad + \int_0^1 (\sum \lambda_n \partial_{S_n} F S'_n) \cdot dx \\ &= \frac{1}{2} \int_0^1 \left(Ric_{\bar{H}\gamma} \langle (\mathbf{ADF})(s)(\gamma) \rangle + \hat{\Theta}_{\bar{H}\gamma} \langle (\mathbf{ADF})(s)(\gamma) \rangle \right) \cdot dx_s \\ &\quad + \int_0^1 (\mathbf{ADF})'(s)(\gamma) \cdot dx_s \end{aligned}$$

ν -a.e., where $\gamma = I(x) \in \mathbf{P}_{m_0}(M)$ for μ -a.e. $x \in X$.

5. LOCAL SECOND MOMENT

We begin this section with a general lemma. For a moment, we introduce a new setting more general than the situation in Sections 1-4. Let E be a Hausdorff topological space and let $\mathcal{B}(E)$ denote its Borel σ -algebra. Suppose, furthermore, that $\mathcal{B}(E) = \sigma(C(E))$ where $C(E)$ denotes the set of all continuous functions on E . Let ν be a probability measure on (E, \mathcal{B}) and $(\mathcal{E}, D(\mathcal{E}))$ a quasi-regular Dirichlet form on $L^2(E, \nu)$. Let $(A, D(A))$ denote the generator of $(\mathcal{E}, D(\mathcal{E}))$ and assume that there exists a subspace $\mathbb{G} \subseteq D(A)$, dense in $(D(\mathcal{E}), \mathcal{E}_1^{1/2})$, such that $g \in \mathbb{G}$ implies that $g^2 \in D(A)$. Then, according to [3], Proposition I.4.1.3 and Corollary I.4.2.3, there exists a unique carré du champ operator $\Gamma : D(\mathcal{E}) \times D(\mathcal{E}) \rightarrow L^1(E, \nu)$. In particular, for all $f, g \in D(\mathcal{E}) \cap L^\infty(E, \nu)$, it holds that

$$(5.1) \quad \int g \Gamma(f, f) d\nu = -\mathcal{E}(g, f^2) + 2\mathcal{E}(fg, f).$$

Let $\mathbf{M} = (\Omega, \mathcal{F}, (X_t)_{t \geq 0}, (P_x)_{x \in E})$ denote the associated right process, set $\mathbf{1}(x) = 1$, $x \in E$, and define

$$T_t f(x) := \int f(y) P_x(X_t \in dy), \quad t \geq 0, \quad x \in E, \quad f \in L^2(E, \nu).$$

Proposition 5.1. *Suppose $\mathbf{1} \in \mathbb{G}$, $T_t \mathbf{1} = \mathbf{1}$ for all $t \geq 0$, and $fg \in D(\mathcal{E}) \cap L^\infty(E, \nu)$ if $f \in D(\mathcal{E}) \cap L^\infty(E, \nu)$ and $g \in \mathbb{G}$. Then we have*

$$\begin{aligned} \lim_{t \rightarrow 0} \frac{1}{t} \int (f(y) - f(\cdot))^2 P_\cdot(X_t \in dy) \\ = \Gamma(f, f) \quad \text{weakly in } L^1(E, \nu), \quad f \in D(\mathcal{E}) \cap L^\infty(E, \nu). \end{aligned}$$

Proof. Step 1. Let Id denote the identity in $L^2(E, \nu)$ and let $f \in D(\mathcal{E}) \cap L^\infty(E, \nu)$ and $g \in \mathbb{G}$. Since $g \in D(A) \subseteq L^2(E, \nu)$, $f^2 \in L^\infty(E, \nu) \subseteq L^2(E, \nu)$, and $T_t - Id$ is a selfadjoint operator in $L^2(E, \nu)$, $t > 0$, we can conclude

$$\begin{aligned} -\mathcal{E}(g, f^2) &= (Ag, f^2)_{L^2(E, \nu)} \\ &= \left(\lim_{t \rightarrow 0} \frac{(T_t - Id)g}{t}, f^2 \right)_{L^2(E, \nu)} \\ &= \lim_{t \rightarrow 0} \left(\frac{(T_t - Id)g}{t}, f^2 \right)_{L^2(E, \nu)} \\ (5.2) \quad &= \lim_{t \rightarrow 0} \left(g, \frac{(T_t - Id)f^2}{t} \right)_{L^2(E, \nu)}. \end{aligned}$$

Let $(E_\lambda)_{\lambda \geq 0}$ denote the (right continuous) resolution of the identity with respect to $-A$, i.e.,

$$-Af = \int_{[0, \infty)} \lambda dE_\lambda f, \quad f \in D(A),$$

recall that A is a nonpositive definite selfadjoint operator in $L^2(E, \nu)$. The closed form $(\mathcal{E}, D(\mathcal{E}))$ has, therefore, a representation

$$\mathcal{E}(f, f) = \int_{[0, \infty)} \lambda d\|E_\lambda f\|_{L^2(E, \nu)}^2, \quad f \in D(\mathcal{E}).$$

Taking into consideration $-\lambda \leq \frac{e^{-\lambda t} - 1}{t} \leq 0$ if $\lambda \geq 0$ and $t > 0$, we obtain

$$\begin{aligned} \mathcal{E}(fg, f) &= \int_{[0, \infty)} \lambda d(E_\lambda(fg), E_\lambda f)_{L^2(E, \nu)} \\ &= -\lim_{t \rightarrow 0} \int_{[0, \infty)} \frac{e^{-\lambda t} - 1}{t} d(E_\lambda(fg), E_\lambda f)_{L^2(E, \nu)} \\ &= -\lim_{t \rightarrow 0} \left(\int_{[0, \infty)} dE_\lambda(fg), \int_{[0, \infty)} \frac{e^{-\lambda t} - 1}{t} dE_\lambda f \right)_{L^2(E, \nu)} \\ (5.3) \quad &= -\lim_{t \rightarrow 0} \left(fg, \frac{T_t f - f}{t} \right)_{L^2(E, \nu)}. \end{aligned}$$

Combining $T_t \mathbf{1} = \mathbf{1}$, $t \geq 0$, (5.1), (5.2), and (5.3), we verify

$$\begin{aligned}
 \int g \Gamma(f, f) d\nu &= -\mathcal{E}(g, f^2) + 2\mathcal{E}(fg, f) \\
 &= \lim_{t \rightarrow 0} \int g \cdot \left(\frac{T_t f^2 - f^2}{t} - 2f \frac{T_t f - f}{t} \right) d\nu \\
 (5.4) \quad &= \lim_{t \rightarrow 0} \int g(x) \cdot \frac{1}{t} \int (f(y) - f(x))^2 P_x(X_t \in dy) \nu(dx).
 \end{aligned}$$

Step 2. Again, let $f \in D(\mathcal{E}) \cap L^\infty(E, \nu)$. For $g = \mathbf{1}$, relation (5.4) reduces to

$$(5.5) \quad 2\mathcal{E}(f, f) = \lim_{t \rightarrow 0} \left\| \frac{1}{t} \int (f(y) - f(\cdot))^2 P(X_t \in dy) \right\|_{L^1(E, \nu)}.$$

Setting

$$\varphi_t := \begin{cases} \Gamma(f, f) & \text{if } t = 0, \\ \frac{1}{t} \int (f(y) - f(\cdot))^2 P(X_t \in dy) & \text{if } t > 0 \end{cases}$$

from $\varphi_t = \frac{T_t f^2 - f^2}{t} - 2f \frac{T_t f - f}{t}$, $t > 0$, and (5.5) it follows that $\|\varphi_t\|_{L^1(E, \nu)}$ is continuous on $[0, \infty)$ and $\lim_{t \rightarrow \infty} \|\varphi_t\|_{L^1(E, \nu)} = 0$. Thus, the family $(\varphi_t)_{t \geq 0}$ is uniformly bounded in $L^1(E, \nu)$. Now the statement of the lemma is a consequence of relation (5.4). \square

In the remainder of this section, we follow the setting of Sections 1-4. In particular, let $\mathbf{M} = (\Omega, \mathcal{F}, (X_t)_{t \geq 0}, (P_\gamma)_{\gamma \in \mathbf{P}_{m_0}(M)})$ denote the right process associated with $(\mathcal{E}, D(\mathcal{E}))$; cf. Theorem 3.5. Furthermore, recall that $x^v(p)$, $v \in \{1, \dots, N\}$, denote the standard coordinates of $p \in M$ embedded in \mathbb{R}^N . For fixed $s \in [0, 1]$ and $v \in \{1, \dots, N\}$, introduce the function x_s^v by $x_s^v(\gamma) := x^v(\gamma(s))$, $\gamma \in \mathbf{P}_{m_0}(M)$.

Lemma 5.2. *Suppose the validity of relation (3.3), i.e.,*

$$\lambda_i \leq c i^{1-\varepsilon}, \quad i \in \mathbb{N}, \quad \text{for some } c > 0 \text{ and } \varepsilon \in (0, 1).$$

Then $x_s^v \in D(\mathcal{E}) \cap L^\infty(\nu)$ and there exists $C_6 > 0$ such that

$$(5.6) \quad \mathcal{E}(x_s^v, x_s^v) \leq C_6$$

for all $s \in [0, 1]$ and $v \in \{1, \dots, N\}$.

Proof. Let $v \in \{1, \dots, N\}$, $s \in [0, 1]$, and $s_n \in \{\frac{l}{2^m} : l \in \{1, \dots, 2^m\}, m \in \mathbb{N}\}$, $n \in \mathbb{N}$, be a sequence with $s_n \xrightarrow{n \rightarrow \infty} s$. Then

$$(5.7) \quad x_{s_n}^v \xrightarrow{n \rightarrow \infty} x_s^v \quad \nu\text{-a.e.}$$

Furthermore, as M is compact, there is a constant $C_7 > 0$ such that

$$(5.8) \quad \|x_{s_n}^v\|_{L^2(\nu)}^2 \leq C_7,$$

independent of $n \in \mathbb{N}$. As in (3.5)-(3.8), it follows from (3.3) that

$$(5.9) \quad \mathcal{E}(x_{s_n}^v, x_{s_n}^v) \leq C_1,$$

independent of $n \in \mathbb{N}$, where C_1 is the constant introduced in (3.7). Now, the above mentioned Banach-Saks property of the Hilbert space $(D(\mathcal{E}), \mathcal{E}_1^{1/2})$ and (5.7)-(5.9) imply $x_s^v \in D(\mathcal{E}) \cap L^\infty(\nu)$ and from the closedness of $(\mathcal{E}, D(\mathcal{E}))$ on $L^2(\nu)$, relation (5.6) with $C_6 := C_1 + C_7$ can be derived. \square

Set $\mathbb{G} := Y$. According to Theorems 2.2 and 4.2, \mathbb{G} is dense in $(D(\mathcal{E}), \mathcal{E}_1^{1/2})$ and we have $\mathbb{G} \subseteq D(A)$. Obviously, $g \in \mathbb{G}$ implies $g^2 \in \mathbb{G}$. Thus, there exists a carré du champ operator Γ and we have (5.1). In order to formulate the following theorem, we notice that, for $h \in L^\infty(\nu)$,

$$\begin{aligned} \int |h| \cdot \Gamma(x_s^v, x_s^v) d\nu &\leq 2\|h\|_{L^\infty(\nu)} \cdot \mathcal{E}(x_s^v, x_s^v) \\ &\leq 2C_6\|h\|_{L^\infty(\nu)}, \end{aligned}$$

independent of $s \in [0, 1]$ and $v \in \{1, \dots, N\}$ (cf. (5.6)), which implies

$$(5.10) \quad \int_{\gamma \in \mathbf{P}_{m_0}} |h| \cdot \sum_{v=1}^N \int_{s \in [0, 1]} \Gamma(x_s^v(\gamma), x_s^v(\gamma)) ds \nu(d\gamma) \leq 2NC_6\|h\|_{L^\infty(\nu)}.$$

Theorem 5.3. *Suppose that relation (3.3) is valid, i.e.,*

$$\lambda_i \leq ci^{1-\varepsilon}, \quad i \in \mathbb{N}, \quad \text{for some } c > 0 \text{ and } \varepsilon \in (0, 1).$$

For all $h \in L^\infty(\nu)$, we have

$$\begin{aligned} &\lim_{t \rightarrow 0} \int h(\tau) \cdot \frac{1}{t} \int \sum_{v=1}^N \|x^v(\gamma) - x^v(\tau)\|_{L^2([0, 1], ds)}^2 P_\tau(X_t \in d\gamma) \nu(d\tau) \\ (5.11) \quad &= \int h(\tau) \cdot \sum_{v=1}^N \int_{s \in [0, 1]} \Gamma(x_s^v, x_s^v) ds \nu(d\tau). \end{aligned}$$

Proof. In order to apply Proposition 5.1, we note that $\mathbf{1} \in \mathbb{G}$, $P_\tau(X_t \in \mathbf{P}_{m_0}(M)) = 1$, $t \geq 0$, $\tau \in \mathbf{P}_{m_0}(M)$, and that because of $\mathbb{G} = Y \subseteq D(\mathcal{E}) \cap L^\infty(\nu)$, we have $fg \in D(\mathcal{E}) \cap L^\infty(\nu)$ if $f \in D(\mathcal{E}) \cap L^\infty(\nu)$ and $g \in \mathbb{G}$.

By virtue of Lemma 5.2 and Proposition 5.1, for all $s \in [0, 1]$, $v \in \{1, \dots, N\}$, and $h \in L^\infty(\nu)$, it holds that

$$\lim_{t \rightarrow 0} \int h \cdot \frac{1}{t} \int (x_s^v(\gamma) - x_s^v)^2 P(X_t \in d\gamma) d\nu = \int h \Gamma(x_s^v, x_s^v) d\nu.$$

Since $\|\frac{1}{t} \int (x_s^v(\gamma) - x_s^v)^2 P(X_t \in d\gamma)\|_{L^1(\nu)}$ is bounded for $t > 0$ (cf. proof of Proposition 5.1), it can be concluded from dominated convergence that, for all $v \in \{1, \dots, N\}$,

$$\begin{aligned} &\lim_{t \rightarrow 0} \int_{s \in [0, 1]} \int h \cdot \frac{1}{t} \int (x_s^v(\gamma) - x_s^v)^2 P(X_t \in d\gamma) d\nu ds \\ &= \int_{s \in [0, 1]} \int h \Gamma(x_s^v, x_s^v) d\nu ds, \quad h \in L^\infty(\nu). \end{aligned}$$

Relation (5.11) is now a direct consequence of (5.10) and Fubini's theorem. \square

Remark. (6) Condition (3.3) in Proposition 3.3, Lemma 5.2, and Theorem 5.3 can be weakened. Recalling (3.6) it turns out that it is sufficient to require

$$\sum_{m=1}^{\infty} \lambda_d 2^m 2^{-m} < \infty$$

instead of (3.3).

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DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE, UNIVERSITY OF JENA, D-07740 JENA, GERMANY

Current address: Department of Mathematical Sciences, University of Delaware, 501 Ewing Hall, Newark, Delaware 19716-2553

E-mail address: loebus@math.udel.edu